Pascal Contest

(Grade 9)

Tuesday, February 28, 2017
(in North America and South America)

Wednesday, March 1, 2017
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes   ©2016 University of Waterloo

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name and city/town in the box in the upper right corner.

5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**

6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

8. Diagrams are *not* drawn to scale. They are intended as aids only.

9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{4 \times 3}{2 + 1}$ is

    (A) 4 **(B)** 7 **(C)** 3 **(D)** 6 **(E)** 5

2. In the diagram, how many 1×1 squares are shaded in the 6×6 grid?

    (A) 29 **(B)** 30 **(C)** 31

    (D) 32 **(E)** 33
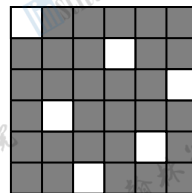
3. In the diagram, the ratio of the number of shaded triangles to the number of unshaded triangles is

    (A) $5 : 2$ **(B)** $5 : 3$ **(C)** $8 : 5$

    (D) $5 : 8$ **(E)** $2 : 5$
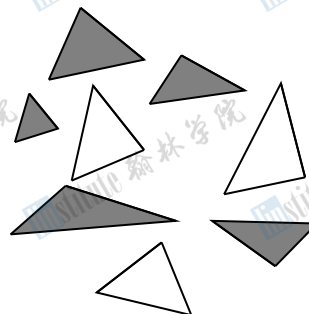
4. Which of the following is closest in value to 7?

    (A) $\sqrt{70}$ **(B)** $\sqrt{60}$ **(C)** $\sqrt{50}$ **(D)** $\sqrt{40}$ **(E)** $\sqrt{80}$

5. Kamal turned his computer on at 2 p.m. on Friday. He left his computer on for exactly 30 consecutive hours. At what time did he turn his computer off?

    (A) 4 p.m. on Saturday
    (B) 6 p.m. on Saturday
    (C) 8 p.m. on Sunday
    (D) 6 p.m. on Sunday
    (E) 8 p.m. on Saturday

6. At six different times on Canada Day in 2016, the number of people at the Pascal Zoo were counted. The graph to the right shows these results. During which of the following periods did the number of people at the zoo have the largest increase?

    (A) 9:00 a.m. to 10:00 a.m.
    (B) 10:00 a.m. to 11:00 a.m.
    (C) 11:00 a.m. to 12:00 p.m.
    (D) 12:00 p.m. to 1:00 p.m.
   **(E)** 1:00 p.m. to 2:00 p.m.
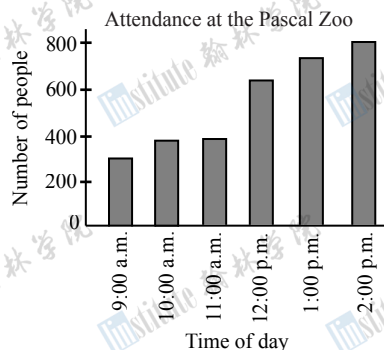
7. If $2x - 3 = 10$, what is the value of $4x$?

    (A) 23   **(B)** 24   **(C)** 28   **(D)** 26   **(E)** 20

8. Three integers from the list $1, 2, 4, 8, 16, 20$ have a product of 80. What is the sum of these three integers?

    (A) 21   **(B)** 22   **(C)** 25   **(D)** 29   **(E)** 26

9. Wally makes a whole pizza and shares it with three friends. Jovin takes $\frac{1}{3}$ of the pizza, Anna takes $\frac{1}{6}$ of the pizza, and Olivia takes $\frac{1}{4}$ of the pizza. What fraction of the pizza is left for Wally?

    (A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{10}{13}$ **(D)** $\frac{1}{12}$ **(E)** $\frac{1}{3}$

10. Which of the following expressions is equal to an odd integer for every integer n?

    (A) $2017 - 3n$ **(B)** $2017 + n$ **(C)** $2017n$ **(D)** $2017 + n^2$ **(E)** $2017 + 2n$


Part B: Each correct answer is worth 6.

11. Jeff and Ursula each run 30 km. Ursula runs at a constant speed of 10 km/h. Jeff also runs at a constant speed. If Jeff's time to complete the 30 km is 1 hour less than Ursula's time to complete the 30 km, at what speed does Jeff run?

    (A) 6 km/h   **(B)** 11 km/h   **(C)** 12 km/h   **(D)** 15 km/h   **(E)** 22.5 km/h

12. A small square is drawn inside a larger square as shown. The area of the shaded region and the area of the unshaded region are each 18 cm^2. What is the side length of the larger square?

    (A) 3 cm   **(B)** 4 cm   **(C)** 6 cm

    (D) 9 cm   **(E)** 12 cm

13. Janet picked a number, added 7 to the number, multiplied the sum by 2, and then subtracted 4. If the final result was 28, what number did Janet pick?

    (A) 9   **(B)** 5   **(C)** 19   **(D)** 23   **(E)** 11

14. Tobias downloads m apps. Each app costs \$2.00 plus 10% tax. He spends \$52.80 in total on these m apps. What is the value of m?

    (A) 20   **(B)** 22   **(C)** 18   **(D)** 24   **(E)** 26

15. In the diagram, the side lengths of four squares are shown. The area of the fifth square is k. What is the value of k?

    (A) 64   **(B)** 49   **(C)** 36

    (D) 25   **(E)** 16
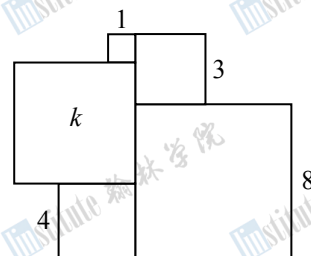
16. A circular spinner is divided into six regions, as shown. Four regions each have a central angle of $x°$. The remaining regions have central angles of $20°$ and $140°$. An arrow is attached to the centre of the circle. The arrow is spun once. What is the probability that the arrow stops on a shaded region?
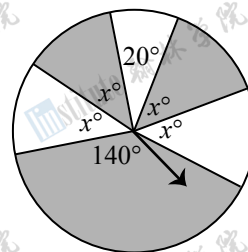
(A) $\frac{2}{3}$ (B) $\frac{7}{8}$ (C) $\frac{1}{2}$

(D) $\frac{5}{12}$ (E) $\frac{7}{12}$

17. Igor is shorter than Jie. Faye is taller than Goa. Jie is taller than Faye. Han is shorter than Goa. Who is the tallest?

(A) Faye   (B) Goa   (C) Han   (D) Igor   (E) Jie

18. Given two different numbers on a number line, the number to the right is greater than the number to the left. The positions of x, x^3 and x^2 are marked on a number line. Which of the following is a possible value of x?
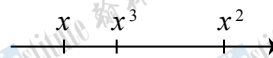
(A) $\frac{1}{5}$ (B) $\frac{3}{2}$ (C) $-\frac{2}{5}$

(D) $-\frac{4}{3}$ (E) 2

19. In the diagram, M is the midpoint of YZ, $\angle XMZ = 30°$, and $\angle XYZ = 15°$. The measure of $\angle XZY$ is

(A) $75°$ (B) $65°$ (C) $60°$

(D) $80°$ (E) $85°$
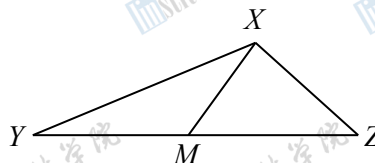
20. A solid cube is made of white plastic and has dimensions $n \times n \times n$, where n is a positive integer larger than 1. The six faces of the cube are completely covered with gold paint. This cube is then cut into n^3 cubes, each of which has dimensions $1 \times 1 \times 1$. Each of these $1 \times 1 \times 1$ cubes has 0, 1, 2, or 3 gold faces. The number of $1 \times 1 \times 1$ cubes with 0 gold faces is strictly greater than the number of $1 \times 1 \times 1$ cubes with exactly 1 gold face. What is the smallest possible value of n?

(A) 7   (B) 8   (C) 9   (D) 10   (E) 4

Part C: Each correct answer is worth 8.

21. Each of the numbers $1, 5, 6, 7, 13, 14, 17, 22, 26$ is placed in a different circle below. The numbers 13 and 17 are placed as shown.


Jen calculates the average of the numbers in the first three circles, the average of the numbers in the middle three circles, and the average of the numbers in the last three circles. These three averages are equal. What number is placed in the shaded circle?

(A) 1   (B) 5   (C) 6   (D) 7   (E) 14

22. In the diagram, $UVWX$ is a rectangle that lies flat on a horizontal floor. A vertical semi-circular wall with diameter XW is constructed. Point Z is the highest point on this wall. If $UV = 20$ and $VW = 30$, the perimeter of $\triangle UVZ$ is closest to

   (A) 95   **(B)** 86   **(C)** 102

   (D) 83   **(E)** 92
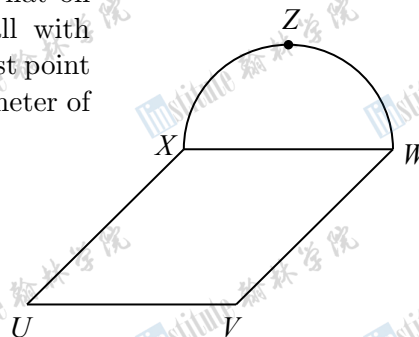
23. An *Anderson number* is a positive integer k less than $10\,000$ with the property that k^2 ends with the digit or digits of k. For example, 25 is an Anderson number because 625 ends with 25, but 75 is not an Anderson number because 5625 does not end with 75. If S is the sum of all *even* Anderson numbers, what is the sum of the digits of S?

   (A) 17   **(B)** 18   **(C)** 11   **(D)** 33   **(E)** 24

24. A town has 2017 houses. Of these 2017 houses, 1820 have a dog, 1651 have a cat, and 1182 have a turtle. If x is the largest possible number of houses that have a dog, a cat, and a turtle, and y is the smallest possible number of houses that have a dog, a cat, and a turtle, then $x - y$ is

   (A) 1182   **(B)** 638   **(C)** 563   **(D)** 619   **(E)** 466

25. Sam thinks of a 5-digit number. Sam's friend Sally tries to guess his number. Sam writes the number of matching digits beside each of Sally's guesses. A digit is considered "matching" when it is the correct digit in the correct position.

Guess	Number of Matching Digits
51545	2
21531	1
71794	0
59135	1
58342	2
37348	2
71744	1

   What is the sum of all of the possibilities for Sam's number?

   (A) 525 768   **(B)** 527 658   **(C)** 527 568   **(D)** 526 578   **(E)** 526 758

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- Information about careers in and applications of mathematics and computer science

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