The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Tuesday, February 28, 2017
(in North America and South America)

Wednesday, March 1, 2017
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

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Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

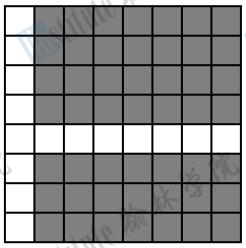
Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $6 \times 2017 - 2017 \times 4$ is

(A) 2 **(B)** 20170 **(C)** 0 **(D)** 4034 **(E)** 24

2. In the diagram, how many 1×1 squares are shaded in the 8×8 grid?

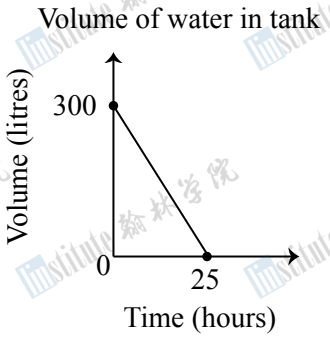


(A) 53 **(B)** 51 **(C)** 47 **(D)** 45 **(E)** 49

3. Three different numbers from the list 2, 3, 4, 6 have a sum of 11. What is the product of these numbers?

(A) 24 **(B)** 72 **(C)** 36 **(D)** 48 **(E)** 32

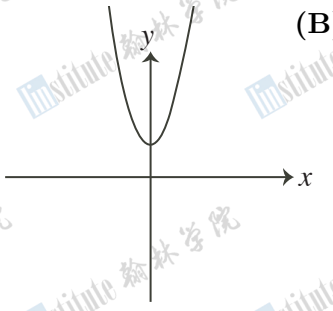
4. The graph shows the volume of water in a 300 L tank as it is being drained at a constant rate. At what rate is the water leaving the tank, in litres per hour?



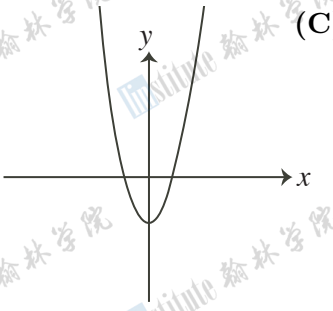
(A) 12 **(B)** 20 **(C)** 2.5 **(D)** 5 **(E)** 15

5. A sketch of $y = -2x^2 + 4$ could be

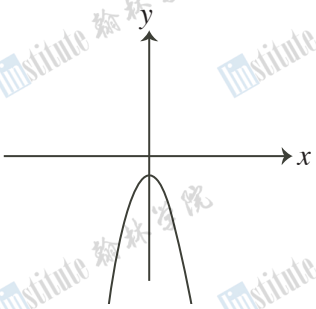
(A)



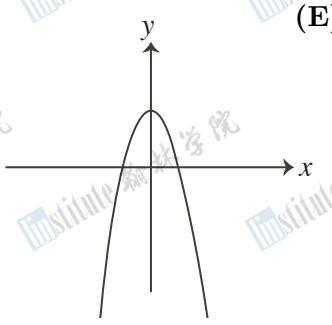
(B)



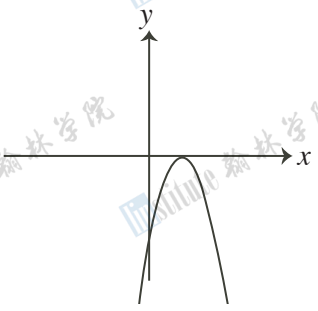
(C)



(D)



(E)



6. Emilia writes down the numbers 5, x and 9. Valentin calculates the mean (average) of each pair of these numbers and obtains 7, 10 and 12. The value of x is

(A) 5 **(B)** 15 **(C)** 3 **(D)** 25 **(E)** 1

7. If $x = 1$ is a solution of the equation $x^2 + ax + 1 = 0$, then the value of a is

(A) 3 **(B)** -1 **(C)** 1 **(D)** 2 **(E)** -2

8. If $\frac{1}{2n} + \frac{1}{4n} = \frac{3}{12}$, then n equals

(A) 6 **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{3}$ **(D)** 2 **(E)** 3

9. Kamile turned her computer off at 5 p.m. Friday, at which point it had been on for exactly 100 hours. At what time had Kamile turned her computer on?

(A) 1 p.m. Tuesday **(B)** 9 p.m. Monday **(C)** 2 p.m. Tuesday
(D) 1 p.m. Monday **(E)** 9 p.m. Wednesday

10. The sum of four different positive integers is 100. The largest of these four integers is n. The smallest possible value of n is

(A) 26 **(B)** 50 **(C)** 28 **(D)** 27 **(E)** 94

Part B: Each correct answer is worth 6.

11. Last Thursday, each of the students in M. Fermat's class brought one piece of fruit to school. Each brought an apple, a banana, or an orange. In total, 20% of the students brought an apple and 35% brought a banana. If 9 students brought oranges, how many students were in the class?

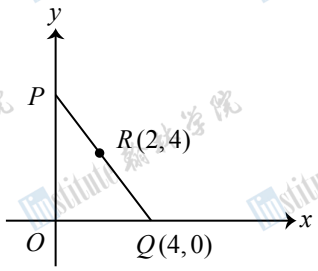
(A) 18 **(B)** 64 **(C)** 24 **(D)** 20 **(E)** 40

12. Digits are placed in the two boxes of $2\square\square$, with one digit in each box, to create a three-digit positive integer. In how many ways can this be done so that the three-digit positive integer is larger than 217?

(A) 81 **(B)** 82 **(C)** 83 **(D)** 92 **(E)** 93

13. In the diagram, P lies on the y-axis, Q has coordinates $(4, 0)$, and PQ passes through the point $R(2, 4)$. What is the area of $\triangle OPQ$?

(A) 8 **(B)** 12 **(C)** 32
(D) 24 **(E)** 16



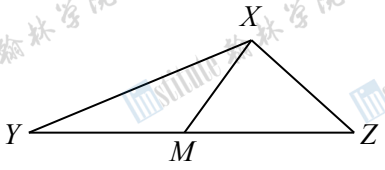
14. The expression

$$\left(1+\tfrac{1}{2}\right)\left(1+\tfrac{1}{3}\right)\left(1+\tfrac{1}{4}\right)\left(1+\tfrac{1}{5}\right)\left(1+\tfrac{1}{6}\right)\left(1+\tfrac{1}{7}\right)\left(1+\tfrac{1}{8}\right)\left(1+\tfrac{1}{9}\right)$$

is equal to

(A) 5 **(B)** $\frac{10}{9}$ **(C)** 9 **(D)** $9\frac{1}{8}$ **(E)** $\frac{1}{2}$

15. In the diagram, M is the midpoint of YZ, $\angle XMZ = 30°$, and $\angle XYZ = 15°$. The measure of $\angle XZY$ is



(A) 75° **(B)** 65° **(C)** 60° **(D)** 80° **(E)** 85°

16. If $x + 2y = 30$, the value of $\frac{x}{5} + \frac{2y}{3} + \frac{2y}{5} + \frac{x}{3}$ is

(A) 8 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 30

17. Aaron has 144 identical cubes, each with edge length 1 cm. He uses all of the cubes to construct a solid rectangular prism, which he places on a flat table. If the perimeter of the base of the prism is 20 cm, what is the sum of all possible heights of the prism?

(A) 31 cm **(B)** 25 cm **(C)** 15 cm **(D)** 22 cm **(E)** 16 cm

18. For any positive real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. For example, $\lfloor 4.2 \rfloor = 4$ and $\lfloor 3 \rfloor = 3$. If $\lfloor x \rfloor \cdot x = 36$ and $\lfloor y \rfloor \cdot y = 71$ where $x, y > 0$, then $x + y$ equals

(A) $\frac{107}{8}$ **(B)** $\frac{119}{8}$ **(C)** $\frac{125}{9}$ **(D)** $\frac{107}{6}$ **(E)** $\frac{101}{7}$

19. A point is *equidistant from the coordinate axes* if the vertical distance from the point to the x-axis is equal to the horizontal distance from the point to the y-axis. The point of intersection of the vertical line $x = a$ with the line with equation $3x+8y = 24$ is equidistant from the coordinate axes. What is the sum of all possible values of a?

(A) 0 **(B)** $-\frac{144}{55}$ **(C)** $-\frac{11}{5}$ **(D)** $\frac{24}{11}$ **(E)** 8

20. If m and n are positive integers with $n > 1$ such that $m^n = 2^{25} \times 3^{40}$, then $m + n$ is

(A) 209 962 **(B)** 1954 **(C)** 209 957 **(D)** 6598 **(E)** 1 049 760

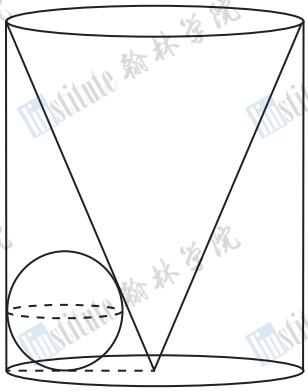
Part C: Each correct answer is worth 8.

21. In the sum shown, each letter represents a different digit with $T \neq 0$ and $W \neq 0$. How many different values of U are possible?

$$\begin{array}{cccccc} & W & X & Y & Z \\ + & W & X & Y & Z \\ \hline T & W & U & Y & V \end{array}$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

22. A cylinder has radius 12 and height 30. The top circular face of the cylinder is the base of a cone and the centre of the bottom circular base of the cylinder is the vertex of the cone. A sphere is placed inside so that it touches the cone, the base of the cylinder and the side of the cylinder as shown. Which of the following is closest to the radius of the sphere?



(A) 4.84 **(B)** 4.74 **(C)** 4.64
(D) 4.54 **(E)** 4.44

23. Sylvia chose positive integers a, b and c.
Peter determined the value of $a + \dfrac{b}{c}$ and got an answer of 101.
Paul determined the value of $\dfrac{a}{c} + b$ and got an answer of 68.
Mary determined the value of $\dfrac{a+b}{c}$ and got an answer of k.
The value of k is
(A) 13 **(B)** 168 **(C)** 152 **(D)** 12 **(E)** 169

24. Eight teams compete in a tournament. Each pair of teams plays exactly one game against each other. There are no ties. If the two possible outcomes of each game are equally likely, what is the probability that every team loses at least one game and wins at least one game?
(A) $\frac{1799}{2048}$ **(B)** $\frac{1831}{2048}$ **(C)** $\frac{1793}{2048}$ **(D)** $\frac{903}{1024}$ **(E)** $\frac{889}{1024}$

25. Let $r = \sqrt{\frac{\sqrt{53}}{2} + \frac{3}{2}}$. There is a unique triple of positive integers (a, b, c) such that

$$r^{100} = 2r^{98} + 14r^{96} + 11r^{94} - r^{50} + ar^{46} + br^{44} + cr^{40}$$

What is the value of $a^2 + b^2 + c^2$?
(A) 11 421 **(B)** 20 229 **(C)** 16 291 **(D)** 15 339 **(E)** 17 115

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For students...

Thank you for writing the 2017 Fermat Contest! Each year, more than 235 000 students from more than 75 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

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- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

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- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
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