The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Tuesday, February 28, 2017
(in North America and South America)

Wednesday, March 1, 2017
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The expression $6 \times 111 - 2 \times 111$ equals

   (A) 222 **(B)** 333 **(C)** 444 **(D)** 555 **(E)** 666

2. The value of $\dfrac{5^2 - 9}{5 - 3}$ is

   (A) 4 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** 8 **(E)** -2

3. A snowman is built by stacking three spheres with their centres aligned vertically. The spheres have radii of 10 cm, 20 cm and 30 cm. How tall is the snowman?

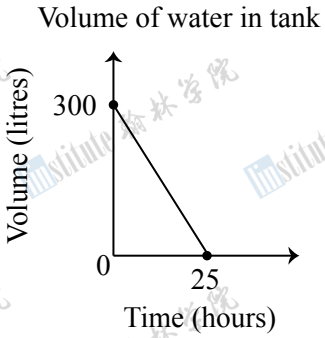
   

   (A) 90 cm **(B)** 100 cm **(C)** 110 cm
   (D) 120 cm **(E)** 130 cm

4. Which of the following fractions has the greatest value?

   (A) $\frac{44444}{55555}$ **(B)** $\frac{5555}{6666}$ **(C)** $\frac{666}{777}$ **(D)** $\frac{77}{88}$ **(E)** $\frac{8}{9}$

5. The graph shows the volume of water in a 300 L tank as it is being drained at a constant rate. At what rate is the water leaving the tank, in litres per hour?

   (A) 12 **(B)** 20 **(C)** 2.5
   (D) 5 **(E)** 15

6. Penelope folds a piece of paper in half, creating two layers of paper. She folds the paper in half again, creating a total of four layers of paper. If she continues to fold the paper in half, which of the following is a possible number of layers that could be obtained?

   (A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

7. The operation $\diamondsuit$ is defined by $a \diamondsuit b = a^2b - ab^2$. The value of $2 \diamondsuit 7$ is

   (A) -140 **(B)** -70 **(C)** 0 **(D)** 70 **(E)** 140

8. Each of three cards is labelled with three numbers. Which of the following groups of three cards has the properties that the first and second cards have exactly one number in common, the first and third cards have exactly one number in common, and the second and third cards have exactly one number in common?

(A) 135 | 367 | 246

(B) 147 | 234 | 245

(C) 234 | 257 | 124

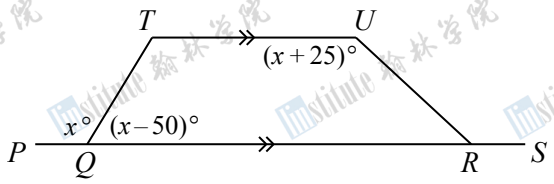
(D) 147 | 234 | 257

(E) 135 | 147 | 235

9. A restaurant bill, including 13% tax but not including a tip, is \$226. The server is paid a tip of 15% based on the bill before tax. How much is the tip that the server is paid?

(A) \$32.87 **(B)** \$29.49 **(C)** \$30.00 **(D)** \$28.00 **(E)** \$44.07

10. In the diagram, TU is parallel to PS and points Q and R lie on PS.
Also, $\angle PQT = x°$, $\angle RQT = (x - 50)°$, and $\angle TUR = (x + 25)°$.

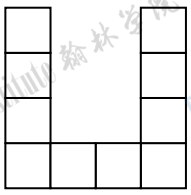


What is the measure of $\angle URS$?

(A) 115° **(B)** 140° **(C)** 135° **(D)** 130° **(E)** 120°

Part B: Each correct answer is worth 6.

11. The figure shown is made up of 10 identical squares. If the area of the figure is 160 cm^2, what is the perimeter of the figure?



(A) 72 cm **(B)** 80 cm **(C)** 88 cm

(D) 64 cm **(E)** 100 cm

12. The mean (average) of the three integers p, q and r is 9.
The mean of the two integers s and t is 14.
The mean of the five integers p, q, r, s, and t is

(A) 11 **(B)** 11.5 **(C)** 12 **(D)** 10 **(E)** 13

13. In the addition shown, each of X, Y and Z represents a digit. What is the value of $X + Y + Z$?

(A) 10 **(B)** 15 **(C)** 22

(D) 20 **(E)** 8

$$\begin{array}{cccc} & X & Y & Z \\ & X & Y & Z \\ + & & Y & Z \\ \hline 1 & 6 & 7 & 5 \end{array}$$

14. Igor is shorter than Jie. Faye is taller than Goa. Jie is taller than Faye. Han is shorter than Goa. Who is the tallest?

(A) Faye **(B)** Goa **(C)** Han **(D)** Igor **(E)** Jie

15. A bag contains red, blue and purple marbles, and does not contain any other marbles. The ratio of the number of red marbles to the number of blue marbles is $4 : 7$. The ratio of the number of blue marbles to the number of purple marbles is $2 : 3$. There are 32 red marbles in the bag. In total, how many marbles are there in the bag?

(A) 162 **(B)** 129 **(C)** 176 **(D)** 164 **(E)** 172

16. If $x + 2y = 30$, the value of $\frac{x}{5} + \frac{2y}{3} + \frac{2y}{5} + \frac{x}{3}$ is

(A) 8 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 30

17. The positive integers r, s and t have the property that $r \times s \times t = 1230$. What is the smallest possible value of $r + s + t$?

(A) 51 **(B)** 52 **(C)** 54 **(D)** 58 **(E)** 53

18. The number of integers n for which $\frac{1}{7} \leq \frac{6}{n} \leq \frac{1}{4}$ is

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 24

19. Two lines with slopes $\frac{1}{4}$ and $\frac{5}{4}$ intersect at $(1, 1)$. What is the area of the triangle formed by these two lines and the vertical line $x = 5$?

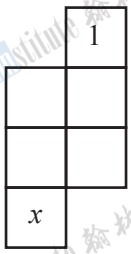
(A) 5 **(B)** 10 **(C)** 8 **(D)** 12 **(E)** 15

20. Car X and Car Y are travelling in the same direction in two different lanes on a long straight highway. Car X is travelling at a constant speed of 90 km/h and has a length of 5 m. Car Y is travelling at a constant speed of 91 km/h and has a length of 6 m. Car Y starts behind Car X and eventually passes Car X. The length of time between the instant when the front of Car Y is lined up with the back of Car X and the instant when the back of Car Y is lined up with the front of Car X is t seconds. The value of t is

(A) 39.6 **(B)** 18.0 **(C)** 21.6 **(D)** 46.8 **(E)** 32.4

Part C: Each correct answer is worth 8.

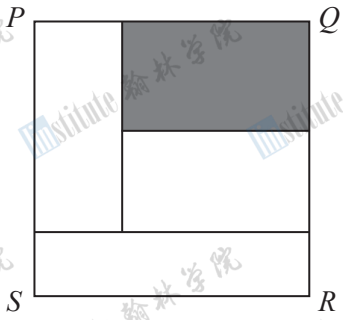
21. The integers 1 to 6 are to be inserted into the grid shown. No two integers that differ by 1 may be in squares that share an edge. If the 1 is inserted as shown, how many different integers can be placed in the box labelled x?



(A) 1 **(B)** 3 **(C)** 5

(D) 0 **(E)** 2

22. In the diagram, square $PQRS$ has side length 42 and is divided into four non-overlapping rectangles. If each of these four rectangles has the same perimeter, what is the area of the shaded rectangle?



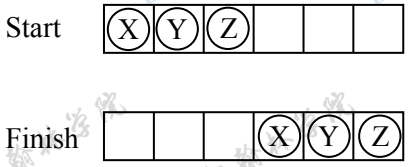
(A) 252 **(B)** 432 **(C)** 441

(D) 490 **(E)** 540

23. The triangle with side lengths 6, 8 and 10 is right-angled, while the triangle with side lengths 6, 8 and 9 is an acute triangle and the triangle with side lengths 6, 8 and 11 is an obtuse triangle. An obtuse triangle with positive area has side lengths 10, 17 and x. If x is an integer, what is the sum of all possible values of x?

(A) 161 **(B)** 198 **(C)** 63 **(D)** 323 **(E)** 224

24. Three coins are placed in the first three of six squares, as shown. A move consists of moving one coin one space to the right, assuming that this space is empty. (No coin can jump over another coin, so the order of the coins will never change.) How many different sequences of moves can be used to move the three coins from the first three squares to the last three squares?



(A) 44 **(B)** 40 **(C)** 42

(D) 48 **(E)** 50

25. A positive integer n with $n \geq 3$ is called a *Nella number* if there exists a positive integer x with $x < n$ and there exists a positive integer m such that

- m is not divisible by x or by $x + 1$, and
- m is divisible by every other positive integer between 1 and n inclusive.

For example, $n = 7$ is a Nella number. How many Nella numbers n are there with $50 \leq n \leq 2017$?

(A) 393 **(B)** 394 **(C)** 395 **(D)** 396 **(E)** 397

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Thank you for writing the 2017 Cayley Contest! Each year, more than 235 000 students from more than 75 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

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- Free copies of past contests
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- Information about careers in and applications of mathematics and computer science

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Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
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