The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Tuesday, February 24, 2015
(in North America and South America)

Wednesday, February 25, 2015
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley or Fermat Contest in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

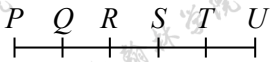
1. The average (mean) of the five numbers 8, 9, 10, 11, 12 is

   (A) 12.5 **(B)** 8 **(C)** 9.6 **(D)** 9 **(E)** 10

2. The value of $\dfrac{2\times 3+4}{2+3}$ is

   (A) 2 **(B)** 5 **(C)** 8 **(D)** 4 **(E)** 11

3. Six points P, Q, R, S, T, U are equally spaced along a straight path. Emily walks from P to U and then back to P. At which point has she completed 70% of her walk?



   (A) T **(B)** Q **(C)** R
   (D) S **(E)** U

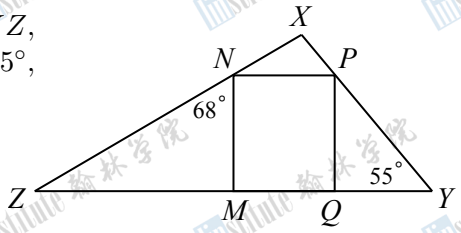
4. If $x=-3$, then $(x-3)^2$ equals

   (A) 12 **(B)** 36 **(C)** -12 **(D)** 0 **(E)** -36

5. The points $P(3,-2)$, $Q(3,1)$, $R(7,1)$, and S form a rectangle. What are the coordinates of S?

   (A) $(-1,-2)$ **(B)** $(7,-2)$ **(C)** $(7,4)$ **(D)** $(3,7)$ **(E)** $(1,-2)$

6. In the diagram, $MNPQ$ is a rectangle with points M, N, P, and Q on the sides of $\triangle XYZ$, as shown. If $\angle ZNM = 68^\circ$ and $\angle XYZ = 55^\circ$, what is the measure of $\angle YXZ$?



   (A) 77° **(B)** 113° **(C)** 93°
   (D) 97° **(E)** 103°

7. Violet has one-half of the money she needs to buy her mother a necklace. After her sister gives her \$30, she has three-quarters of the amount she needs. Violet's father agrees to give her the rest. The amount that Violet's father will give her is

   (A) \$7.50 **(B)** \$15 **(C)** \$22.50 **(D)** \$30 **(E)** \$120

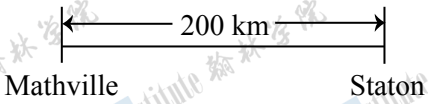
8. If x and y are positive integers with $3^x 5^y = 225$, then $x+y$ equals

   (A) 7 **(B)** 4 **(C)** 5 **(D)** 3 **(E)** 8

9. At Barker High School, a total of 36 students are on either the baseball team, the hockey team, or both. If there are 25 students on the baseball team and 19 students on the hockey team, how many students play both sports?

   (A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

10. Anca and Bruce left Mathville at the same time. They drove along a straight highway towards Staton. Bruce drove at 50 km/h. Anca drove at 60 km/h, but stopped along the way to rest. They both arrived at Staton at the same time. For how long did Anca stop to rest?



(A) 40 minutes **(B)** 10 minutes **(C)** 67 minutes
(D) 33 minutes **(E)** 27 minutes

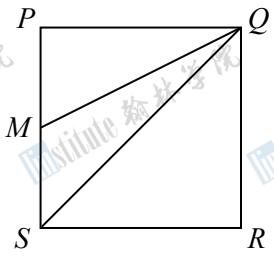
Part B: Each correct answer is worth 6.

11. Three-digit positive integers such as 789 and 998 use no digits other than 7, 8 and 9. In total, how many three-digit positive integers use no digits other than 7, 8 and 9?
(A) 36 **(B)** 6 **(C)** 9 **(D)** 18 **(E)** 27

12. If $\cos 60^\circ = \cos 45^\circ \cos\theta$ with $0^\circ \leq \theta \leq 90^\circ$, then θ equals
(A) 0° **(B)** 15° **(C)** 30° **(D)** 45° **(E)** 60°

13. At the end of the year 2000, Steve had \$100 and Wayne had \$10 000. At the end of each following year, Steve had twice as much money as he did at the end of the previous year and Wayne had half as much money as he did at the end of the previous year. At the end of which year did Steve have more money than Wayne for the first time?
(A) 2002 **(B)** 2003 **(C)** 2004 **(D)** 2005 **(E)** 2006

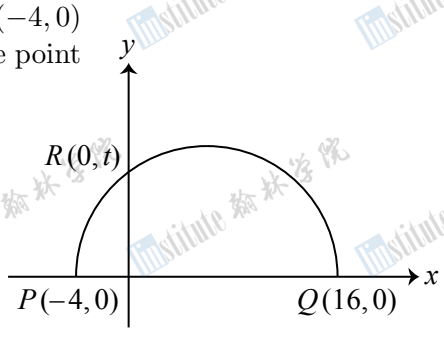
14. In the diagram, $PQRS$ is a square and M is the midpoint of PS. The ratio of the area of $\triangle QMS$ to the area of square $PQRS$ is
(A) 1 : 6 **(B)** 1 : 4 **(C)** 1 : 3
(D) 1 : 8 **(E)** 1 : 2



15. A music test included 50 multiple choice questions. Zoltan's score was calculated by
- adding 4 points for each correct answer,
- subtracting 1 point for each incorrect answer, and
- adding 0 points for each unanswered question.

Zoltan answered 45 of the 50 questions and his score was 135 points. The number of questions that Zoltan answered *incorrectly* is
(A) 9 **(B)** 15 **(C)** 41 **(D)** 40 **(E)** 5

16. In the diagram, the line segment with endpoints $P(-4, 0)$ and $Q(16, 0)$ is the diameter of a semi-circle. If the point $R(0, t)$ is on the circle with $t > 0$, then t is
(A) 6 **(B)** 10 **(C)** 8
(D) 9 **(E)** 7



17. If a and b are two distinct numbers with $\dfrac{a+b}{a-b}=3$, then $\dfrac{a}{b}$ equals

(A) -1 **(B)** 3 **(C)** 1 **(D)** 2 **(E)** 5

18. There are two values of k for which the equation $x^2+2kx+7k-10=0$ has two equal real roots (that is, has exactly one solution for x). The sum of these values of k is

(A) 0 **(B)** -3 **(C)** 3 **(D)** -7 **(E)** 7

19. The y-intercepts of three parallel lines are 2, 3 and 4. The sum of the x-intercepts of the three lines is 36. What is the slope of these parallel lines?

(A) $-\frac{1}{3}$ **(B)** $-\frac{2}{9}$ **(C)** $-\frac{1}{6}$ **(D)** -4 **(E)** $-\frac{1}{4}$

20. For how many integers a with $1 \le a \le 10$ is $a^{2014}+a^{2015}$ divisible by 5?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Part C: Each correct answer is worth 8.

21. Amina and Bert alternate turns tossing a fair coin. Amina goes first and each player takes three turns. The first player to toss a tail wins. If neither Amina nor Bert tosses a tail, then neither wins. What is the probability that Amina wins?

(A) $\frac{21}{32}$ **(B)** $\frac{5}{8}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{11}{16}$ **(E)** $\frac{9}{16}$

22. Three distinct integers a, b and c satisfy the following three conditions:

- $abc = 17\,955$,
- a, b and c form an arithmetic sequence in that order, and
- $(3a+b)$, $(3b+c)$, and $(3c+a)$ form a geometric sequence in that order.

What is the value of $a+b+c$?

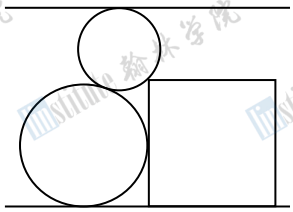
(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, $3, 5, 7$ is an arithmetic sequence with three terms.
A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a non-zero constant. For example, 3, 6, 12 is a geometric sequence with three terms.)

(A) -63 **(B)** -42 **(C)** $-68\,229$ **(D)** -48 **(E)** 81

23. How many pairs (x, y) of non-negative integers with $0 \le x \le y$ satisfy the equation $5x^2-4xy+2x+y^2=624$?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

24. In the diagram, two circles and a square lie between a pair of parallel lines that are a distance of 400 apart. The square has a side length of 279 and one of its sides lies along the lower line. The circles are tangent to each other, and each circle is tangent to one of the lines. Each circle also touches the square at only one point – the lower circle touches a side of the square and the upper circle touches a vertex of the square. If the upper circle has a radius of 65, then the radius of the lower circle is closest to



(A) 151 **(B)** 152 **(C)** 153

(D) 154 **(E)** 155

25. There are F fractions $\frac{m}{n}$ with the properties:

- m and n are positive integers with $m < n$,
- $\frac{m}{n}$ is in lowest terms,
- n is not divisible by the square of any integer larger than 1, and
- the shortest sequence of consecutive digits that repeats consecutively and indefinitely in the decimal equivalent of $\frac{m}{n}$ has length 6.

  (Note: The length of the shortest sequence of consecutive digits that repeats consecutively and indefinitely in $0.12\overline{745} = 0.12745745745745\ldots$ is 3 and the length of the shortest sequence of consecutive digits that repeats consecutively and indefinitely in $0.\overline{5}$ is 1.)

We define $G = F + p$, where the integer F has p digits. What is the sum of the squares of the digits of G?

(A) 170 **(B)** 168 **(C)** 217 **(D)** 195 **(E)** 181

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Thank you for writing the 2015 Fermat Contest! Each year, more than 200 000 students from more than 60 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website cemc.uwaterloo.ca to find

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- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

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