The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Tuesday, February 24, 2015
(in North America and South America)

Wednesday, February 25, 2015
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley or Fermat Contest in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $2 \times 2015 - 2015$ is

(A) 2015 **(B)** 4030 **(C)** 6045 **(D)** 0 **(E)** -2015

2. The expression $\sqrt{1} + \sqrt{9}$ is equal to

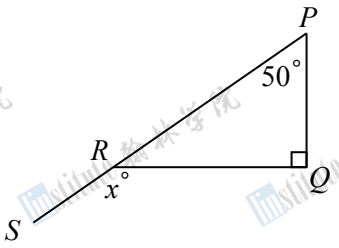
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. The base of a rectangular box measures 2 cm by 5 cm. The volume of the box is 30 cm^3. What is the height of the box?

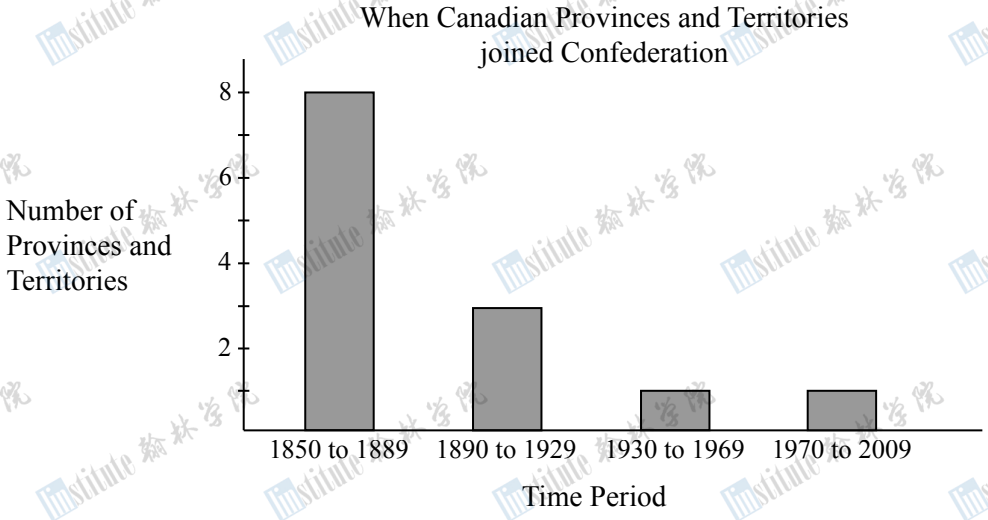
(A) 1 cm **(B)** 2 cm **(C)** 3 cm **(D)** 4 cm **(E)** 5 cm

4. In the diagram, R lies on line segment PS.
The value of x is

(A) 120 **(B)** 130 **(C)** 135
(D) 140 **(E)** 150



5. The bar graph shows the number of provinces and territories that joined Canadian Confederation during each of four 40 year time periods.



If one of the 13 provinces or territories is chosen at random, what is the probability that it joined Canadian Confederation between 1890 and 1969?

(A) $\frac{12}{13}$ **(B)** $\frac{4}{13}$ **(C)** $\frac{5}{13}$ **(D)** $\frac{3}{13}$ **(E)** $\frac{2}{13}$

6. If $a^2 = 9$, then a^4 equals

(A) 27 **(B)** 81 **(C)** 243 **(D)** 729 **(E)** 2187

7. The expression $3 + \frac{1}{10} + \frac{4}{100}$ is *not* equal to

(A) $3\frac{14}{100}$ **(B)** 3.14 **(C)** $3\frac{5}{110}$ **(D)** $3\frac{7}{50}$ **(E)** $\frac{157}{50}$

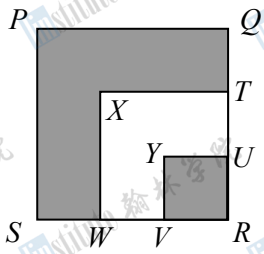
8. Violet has one-half of the money she needs to buy her mother a necklace. After her sister gives her \$30, she has three-quarters of the amount she needs. Violet's father agrees to give her the rest. The amount that Violet's father will give her is

(A) \$7.50 **(B)** \$15 **(C)** \$22.50 **(D)** \$30 **(E)** \$120

9. John goes for a jog every 3 days. He went for a jog on Monday, January 5. He went for his next jog on January 8. What was the date of the next Monday on which he went for a jog?

(A) January 12 **(B)** January 19 **(C)** January 26
(D) February 2 **(E)** February 9

10. In the diagram, square $PQRS$ is 3×3. Points T and U are on side QR with $QT = TU = UR = 1$. Points V and W are on side RS with $RV = VW = WS = 1$. Line segments TX and UY are perpendicular to QR and line segments VY and WX are perpendicular to RS. The ratio of the shaded area to the unshaded area is



(A) 2 : 1 **(B)** 7 : 3 **(C)** 7 : 4
(D) 5 : 4 **(E)** 3 : 1

Part B: Each correct answer is worth 6.

11. The operation $\otimes$ is defined by $a \otimes b = \frac{a}{b} + \frac{b}{a}$. What is the value of $4 \otimes 8$?

(A) $\frac{1}{2}$ **(B)** 1 **(C)** $\frac{5}{4}$ **(D)** 2 **(E)** $\frac{5}{2}$

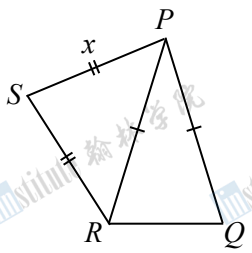
12. The points $(-1, q)$ and $(-3, r)$ are on a line parallel to $y = \frac{3}{2}x + 1$. What is the value of $r - q$?

(A) 3 **(B)** $\frac{4}{3}$ **(C)** $-\frac{3}{4}$ **(D)** $-\frac{4}{3}$ **(E)** -3

13. At Barker High School, a total of 36 students are on either the baseball team, the hockey team, or both. If there are 25 students on the baseball team and 19 students on the hockey team, how many students play both sports?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

14. In the diagram, $\triangle PQR$ is isosceles with $PQ = PR$ and $\triangle PRS$ is isosceles with $PS = SR = x$. Also, the perimeter of $\triangle PQR$ is 22, the perimeter of $\triangle PRS$ is 22, and the perimeter of $PQRS$ is 24. What is the value of x?



(A) 7.5 **(B)** 6.5 **(C)** 7
(D) 6 **(E)** 8

15. If n is a positive integer, the symbol $n!$ (read "n factorial") represents the product of the integers from 1 to n. For example, $4! = (1)(2)(3)(4)$ or $4! = 24$. The ones (units) digit of the sum $1! + 2! + 3! + 4! + 5! + 6! + 7! + 8! + 9! + 10!$ is

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

16. In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum. In the magic square shown, the sum $a + b + c$ equals

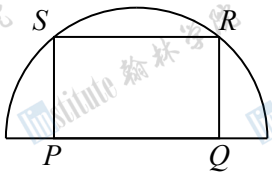
a	13	b
19	c	11
12	d	16

(A) 49 **(B)** 54 **(C)** 47 **(D)** 50 **(E)** 46

17. For the first 30 minutes of a trip, Deanna drove at a constant speed. For the next 30 minutes, she drove at a constant speed 20 km/h faster than her original speed. If the total distance that she travelled was 100 km, how fast did she drive for the first 30 minutes?

(A) 80 km/h **(B)** 90 km/h **(C)** 100 km/h **(D)** 110 km/h **(E)** 120 km/h

18. In the diagram, rectangle $PQRS$ has side PQ on the diameter of the semicircle with R and S on the semicircle. If the diameter of the semicircle is 20 and the length of PQ is 16, then the length of PS is



(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

19. A bank teller has some stacks of bills. The total value of the bills in each stack is \$1000. Every stack contains at least one \$20 bill, at least one \$50 bill, and no other types of bills. If no two stacks have the same number of \$20 bills, what is the maximum possible number of stacks that the teller could have?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 4 **(E)** 8

20. For how many integers n is $72\left(\frac{3}{2}\right)^n$ equal to an integer?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Part C: Each correct answer is worth 8.

21. The average of a list of three consecutive odd integers is 7. When a fourth positive integer, m, different from the first three, is included in the list, the average of the list is an integer. What is the sum of the three smallest possible values of m?

(A) 6 **(B)** 9 **(C)** 21 **(D)** 29 **(E)** 33

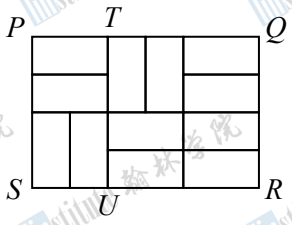
22. Six players compete in a chess tournament. Each player plays exactly two games against every other player. In each game, the winning player earns 1 point and the losing player earns 0 points; if the game results in a draw (tie), each player earns $\frac{1}{2}$ point. What is the minimum possible number of points that a player needs to earn in order to guarantee that he has more points than every other player?

(A) 8 **(B)** $8\frac{1}{2}$ **(C)** 9 **(D)** $9\frac{1}{2}$ **(E)** 10

23. Nylah has her living room lights on a timer. Each evening, the timer switches the lights on randomly at exactly 7:00 p.m., 7:30 p.m., 8:00 p.m., 8:30 p.m., or 9:00 p.m. Later in the evening, the timer switches the lights off at any random time between 11 p.m. and 1 a.m. For example, the lights could be switched on at exactly 7:30 p.m. and off at any one of the infinite number of possible times between 11 p.m. and 1 a.m. On a given night, Nylah's lights are on for t hours. What is the probability that $4 < t < 5$?

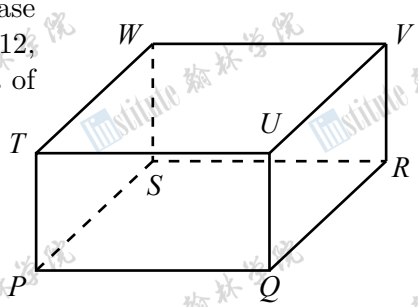
(A) $\frac{1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{3}{10}$ **(E)** $\frac{7}{20}$

24. In the diagram, a rectangular ceiling $PQRS$ measures 6 m by 4 m and is to be completely covered using 12 rectangular tiles, each measuring 1 m by 2 m. If there is a beam, TU, that is positioned so that $PT = SU = 2$ m and that cannot be crossed by any tile, then the number of possible arrangements of tiles is



(A) 180 **(B)** 190 **(C)** 185
(D) 170 **(E)** 175

25. Rectangular prism $PQRSWTUV$ has a square base $PQRS$. Point X is on the face $TUVW$ so that $PX = 12$, $QX = 10$ and $RX = 8$. The maximum possible area of rectangle $PQUT$ is closest to



(A) 67.84 **(B)** 67.82 **(C)** 67.90
(D) 67.86 **(E)** 67.88

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For students...

Thank you for writing the 2015 Cayley Contest! Each year, more than 200 000 students from more than 60 countries register to write the CEMC's Contests.

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