The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Fermat Contest
(Grade 11)

Thursday, February 20, 2014
(in North America and South America)

Friday, February 21, 2014
(outside of North America and South America)

UNIVERSITY OF WATERLOO

WATERLOO MATHEMATICS

Deloitte.

Part A: Each correct answer is worth 5.

1. What is the value of $\dfrac{15 - 3^2}{3}$?

    (A) 2   **(B)** 4   **(C)** 12   **(D)** 48   **(E)** 3

2. The integer 2014 is between

    (A) 10^0 and 10^1 **(B)** 10^1 and 10^2 **(C)** 10^2 and 10^3
    (D) 10^3 and 10^4 **(E)** 10^4 and 10^5

3. If $x = 2$, then $(x + 2 - x)(2 - x - 2)$ equals

    (A) -12 **(B)** 4   **(C)** 0   **(D)** 12   **(E)** -4

4. Two positive integers x and y have $xy = 24$ and $x - y = 5$. The value of $x + y$ is

    (A) 10   **(B)** 11   **(C)** 12   **(D)** 13   **(E)** 14

5. In the diagram, square $WXYZ$ has area 9 and W is at the centre of a circle. If X and Z are on the circle, the area of the circle is

    (A) 3π **(B)** 6π **(C)** 9π

    (D) 18π **(E)** 81π
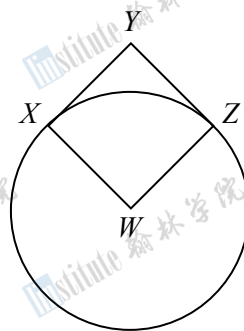
6. If 50% of N is 16, then 75% of N is

    (A) 12   **(B)** 6   **(C)** 20   **(D)** 24   **(E)** 40

7. In the diagram, point T is on side PR of $\triangle PQR$ and QRS is a straight line segment. The value of x is

    (A) 55   **(B)** 70   **(C)** 75

    (D) 60   **(E)** 50
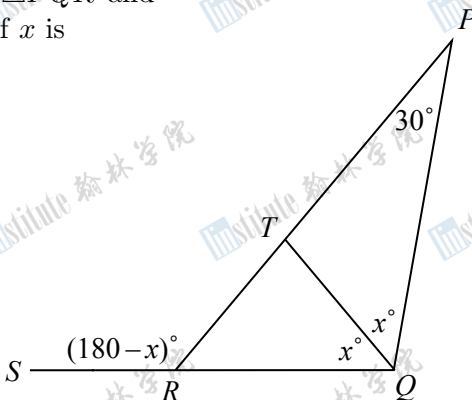
8. In a group of five friends:
   - Amy is taller than Carla.
   - Dan is shorter than Eric but taller than Bob.
   - Eric is shorter than Carla.

   Who is the shortest?

   (A) Amy **(B)** Bob **(C)** Carla **(D)** Dan **(E)** Eric

9. In the diagram, $PQRS$ is a square with side length 8. Points T and U are on PS and QR respectively with $QU = TS = 1$. The length of TU is closest to

   (A) 8.5 **(B)** 9.9 **(C)** 10
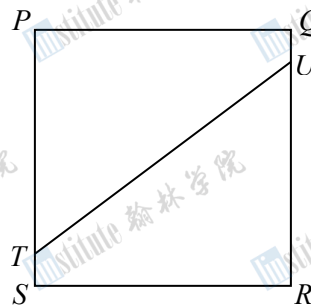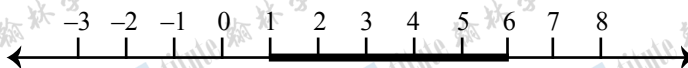
   (D) 10.6 **(E)** 11.3

10. A line segment of length 5 lies along the number line initially between 1 and 6.

    The line segment is rotated 180° about the point at 2. The resulting line segment lies between −2 and 3. This segment is then rotated 180° about the point at 1. The resulting line segment lies between

    (A) −2 and 3 **(B)** −1 and 4 **(C)** 0 and 5 **(D)** −3 and 2 **(E)** −4 and 1

Part B: Each correct answer is worth 6.

11. If $a = \dfrac{2}{3}b$ and $b \neq 0$, then $\dfrac{9a + 8b}{6a}$ is equal to

    (A) $\dfrac{7}{2}$ **(B)** 9 **(C)** $\dfrac{3}{2}$ **(D)** $\dfrac{11}{2}$ **(E)** $\dfrac{17}{6}$

12. If $10^x \cdot 10^5 = 100^4$, what is the value of x?

    (A) 1 **(B)** 35 **(C)** 11 **(D)** $\frac{4}{5}$ **(E)** 3

13. How many positive integers n between 10 and 1000 have the property that the sum of the digits of n is 3?

    (A) 10 **(B)** 8 **(C)** 6 **(D)** 9 **(E)** 7

14. Last summer, Pat worked at a summer camp.
    For each day that he worked, he earned $100 and he was not charged for food.
    For each day that he did not work, he was not paid and he was charged $20 for food.
    After 70 days, the money that he earned minus his food costs equalled $5440.
   On how many of these 70 days did Pat work?

    (A) 60 **(B)** 68 **(C)** 50 **(D)** 57 **(E)** 34

15. On each spin of the spinner shown, the arrow is equally likely to stop on any one of the four numbers. Deanna spins the arrow on the spinner twice. She multiplies together the two numbers on which the arrow stops. Which product is most likely to occur?

    (A) 2   (B) 4   (C) 6

    (D) 8   (E) 12

16. At the start of a 5 hour trip, the odometer in Jill's car indicates that her car had already been driven 13 831 km. The integer 13 831 is a palindrome, because it is the same when read forwards or backwards. At the end of the 5 hour trip, the odometer reading was another palindrome. If Jill never drove faster than 80 km/h, her greatest possible average speed was closest to

    (A) 62 km/h   (B) 20 km/h   (C) 42 km/h   (D) 16 km/h   (E) 77 km/h

17. Sergio recently opened a store. One day, he determined that the average number of items sold per employee to date was 75. The next day, one employee sold 6 items, one employee sold 5 items, and one employee sold 4 items. The remaining employees each sold 3 items. This made the new average number of items sold per employee to date equal to 78.3. How many employees are there at the store?

    (A) 50   (B) 5   (C) 20   (D) 40   (E) 30

18. A square is cut along a diagonal and reassembled to form the parallelogram $PQRS$ as shown in the diagram. If $PR = 90$ mm, what is the area of the original square, in mm^2?
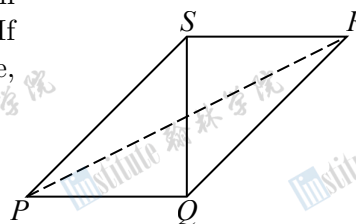
    (A) 324   (B) 1620   (C) 1800

    (D) 2025   (E) 2700

19. Max and Minnie each add up sets of three-digit positive integers. Each of them adds three different three-digit integers whose nine digits are all different. Max creates the largest possible sum. Minnie creates the smallest possible sum. The difference between Max's sum and Minnie's sum is

    (A) 594   (B) 1782   (C) 1845   (D) 1521   (E) 2592

20. In the diagram, $\triangle PQR$ has $PQ = QR = RP = 30$. Points S and T are on PQ and PR, respectively, so that ST is parallel to QR. Points V and U are on QR so that TU is parallel to PQ and SV is parallel to PR. If $VS + ST + TU = 35$, the length of VU is
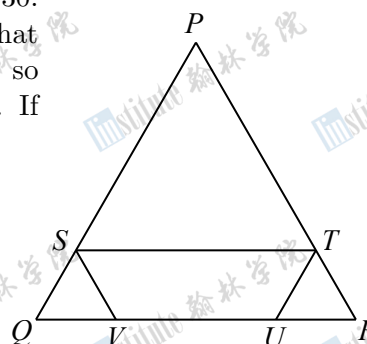
    (A) 21   (B) 15   (C) 18

    (D) 20   (E) 25

Part C: Each correct answer is worth 8.

21. A bin contains 10 kg of peanuts. 2 kg of peanuts are removed and 2 kg of raisins are added and thoroughly mixed in. Then 2 kg of this mixture are removed and 2 kg of raisins are added and thoroughly mixed in again. What is the ratio of the mass of peanuts to the mass of raisins in the final mixture?

    (A) $3:2$ **(B)** $4:1$ **(C)** $5:1$ **(D)** $7:3$ **(E)** $16:9$

22. Jillian drives along a straight road that goes directly from her house (J) to her Grandfather's house (G). Some of this road is on flat ground and some is downhill or uphill. Her car travels downhill at 99 km/h, on flat ground at 77 km/h, and uphill at 63 km/h. It takes Jillian 3 hours and 40 minutes to drive from J to G. It takes her 4 hours and 20 minutes to drive from G to J. The distance between J and G, in km, is

    (A) $318\frac{2}{3}$ **(B)** 324 **(C)** 308 **(D)** $292\frac{3}{5}$ **(E)** $292\frac{1}{9}$

23. $\triangle PQR$ has $PQ = 150$ and $PR = QR = 125$, as shown. Three line segments are drawn parallel to QR, dividing $\triangle PQR$ into four sections of equal area. The height, h, of the bottom section is closest to

    (A) 16.7   **(B)** 16.9   **(C)** 16.5

    (D) 16.3   **(E)** 16.1
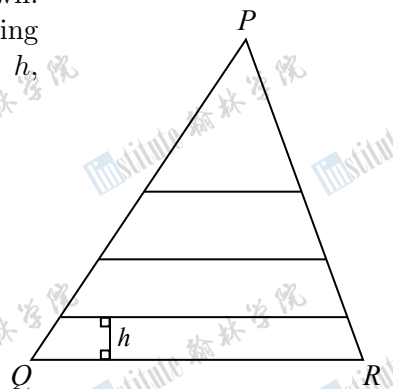
24. Mohammed has eight boxes numbered 1 to 8 and eight balls numbered 1 to 8. In how many ways can he put the balls in the boxes so that there is one ball in each box, ball 1 is not in box 1, ball 2 is not in box 2, and ball 3 is not in box 3?

    (A) $27\,240$ **(B)** $29\,160$ **(C)** $27\,360$ **(D)** $27\,600$ **(E)** $25\,200$

25. Points $P(r, s)$ and $Q(t, u)$ are on the parabola with equation $y = x^2 - \frac{1}{5}mx + \frac{1}{5}n$ so that $PQ = 13$ and the slope of PQ is $\frac{12}{5}$. For how many pairs (m, n) of positive integers with $n \le 1000$ is $r + s + t + u = 27$?

    (A) 28   **(B)** 26   **(C)** 27   **(D)** 29   **(E)** 25

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