



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Thursday, February 20, 2014
(in North America and South America)

Friday, February 21, 2014
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

WATERLOO
MATHEMATICS

Deloitte.

Time: 60 minutes

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Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D, and E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, <http://www.cemc.uwaterloo.ca>. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $2000 - 80 + 200 - 120$ is

(A) 2000 (B) 1600 (C) 2100 (D) 1860 (E) 1760

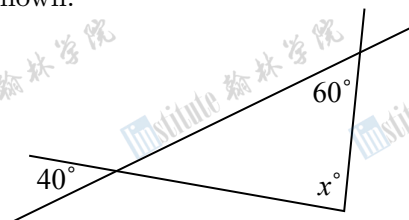
2. If $(2)(3)(4) = 6x$, what is the value of x ?

(A) 2 (B) 6 (C) 12 (D) 3 (E) 4

3. In the diagram, three line segments intersect as shown.

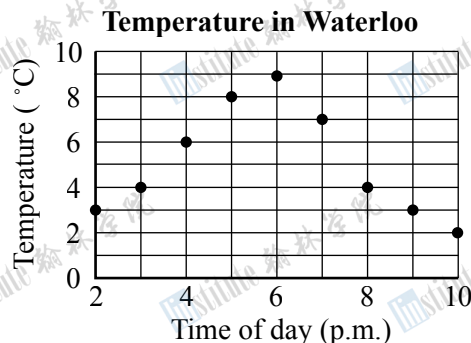
The value of x is

(A) 40 (B) 60 (C) 80
(D) 100 (E) 120



4. At 2 p.m., Sanjay measures the temperature to be 3°C . He measures the temperature every hour after this until 10 p.m. He plots the temperatures that he measures on the graph shown. At what time after 2 p.m. does he again measure a temperature of 3°C ?

(A) 9 p.m. (B) 5 p.m. (C) 8 p.m.
(D) 10 p.m. (E) 7 p.m.



5. If $2n + 5 = 16$, the expression $2n - 3$ equals

(A) 8 (B) 10 (C) 18 (D) 14 (E) 7

6. When the numbers 3 , $\frac{5}{2}$ and $\sqrt{10}$ are listed in order from smallest to largest, the list is

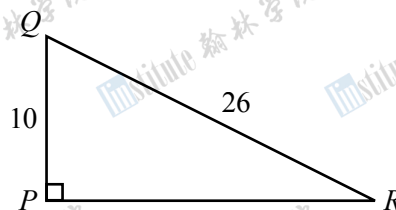
(A) $3, \frac{5}{2}, \sqrt{10}$ (B) $\frac{5}{2}, 3, \sqrt{10}$ (C) $\sqrt{10}, \frac{5}{2}, 3$ (D) $\frac{5}{2}, \sqrt{10}, 3$ (E) $3, \sqrt{10}, \frac{5}{2}$

7. Meg started with the number 100. She increased this number by 20% and then increased the resulting number by 50%. Her final result was

(A) 120 (B) 240 (C) 187.5 (D) 200 (E) 180

8. In the diagram, $\triangle PQR$ has $\angle RPQ = 90^\circ$, $PQ = 10$, and $QR = 26$. The area of $\triangle PQR$ is

(A) 100 (B) 120 (C) 130
(D) 60 (E) 312



9. In a group of five friends:

- Amy is taller than Carla.
- Dan is shorter than Eric but taller than Bob.
- Eric is shorter than Carla.

Who is the shortest?

(A) Amy (B) Bob (C) Carla (D) Dan (E) Eric

10. Consider the following flowchart:



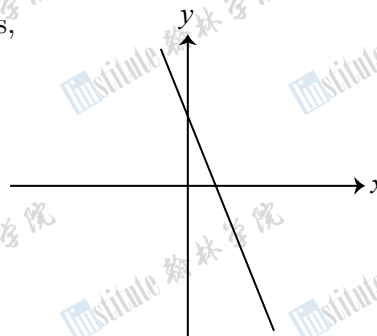
If the OUTPUT is 32, the INPUT must have been

(A) 16 (B) 28 (C) 36 (D) 40 (E) 32

Part B: Each correct answer is worth 6.

11. A line intersects the positive x -axis and positive y -axis, as shown. A possible equation of this line is

(A) $y = 2x + 7$ (B) $y = 4$ (C) $y = -3x - 5$
(D) $y = 5x - 2$ (E) $y = -2x + 3$



12. If $x = 2y$ and $y \neq 0$, then $(x - y)(2x + y)$ equals

(A) $5y^2$ (B) y^2 (C) $3y^2$ (D) $6y^2$ (E) $4y^2$

13. In a factory, Erika assembles 3 calculators in the same amount of time that Nick assembles 2 calculators. Also, Nick assembles 1 calculator in the same amount of time that Sam assembles 3 calculators. How many calculators in total can be assembled by Nick, Erika and Sam in the same amount of time as Erika assembles 9 calculators?

(A) 30 (B) 24 (C) 27 (D) 81 (E) 33

14. Storage space on a computer is measured in gigabytes (GB) and megabytes (MB), where $1 \text{ GB} = 1024 \text{ MB}$. Julia has an empty 300 GB hard drive and puts 300 000 MB of data onto it. How much storage space on the hard drive remains empty?

(A) 72 MB (B) 720 MB (C) 7200 MB (D) 7.2 GB (E) 72 GB

15. In the 4×4 grid shown, each of the four symbols has a different value. The sum of the values of the symbols in each row is given to the right of that row. What is the value of \blacklozenge ?

(A) 5 (B) 6 (C) 7
(D) 8 (E) 9

\heartsuit	\triangle	\triangle	\heartsuit	26
\triangle	\triangle	\triangle	\triangle	24
\square	\blacklozenge	\heartsuit	\blacklozenge	27
\square	\heartsuit	\square	\triangle	33

16. The table shows the results of a poll which asked each student how many hamburgers he or she ate at the end of year class party.

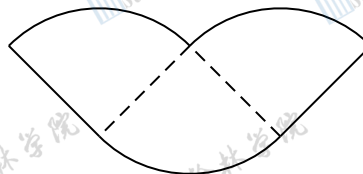
Number of hamburgers	0	1	2	3	4
Number of students	12	14	8	4	2

What is the average (mean) number of hamburgers eaten per student?

(A) 1.8 (B) 2 (C) 1.25 (D) 2.5 (E) 8

17. A circle with area 36π is cut into quarters and three of the pieces are arranged as shown. What is the perimeter of the resulting figure?

(A) $6\pi + 12$ (B) $9\pi + 12$ (C) $9\pi + 18$
(D) $27\pi + 12$ (E) $27\pi + 24$



18. At the post office, Sonita bought some 2¢ stamps and she bought ten times as many 1¢ stamps as 2¢ stamps. She also bought some 5¢ stamps. She did not buy any other stamps. The total value of the stamps that she bought was 100¢. How many stamps did Sonita buy in total?

(A) 66 (B) 30 (C) 44 (D) 63 (E) 62

19. Two different numbers are randomly selected from the set $\{-3, -1, 0, 2, 4\}$ and then multiplied together. What is the probability that the product of the two numbers chosen is 0?

(A) $\frac{1}{10}$ (B) $\frac{1}{5}$ (C) $\frac{3}{10}$ (D) $\frac{2}{5}$ (E) $\frac{1}{2}$

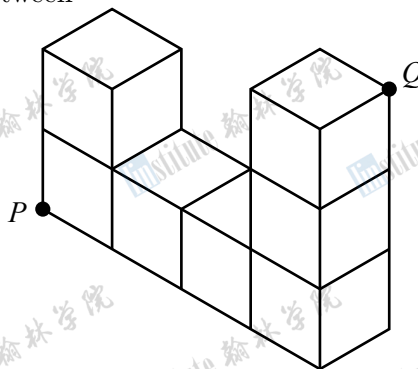
20. If $wxyz$ is a four-digit positive integer with $w \neq 0$, the *layer sum* of this integer equals $wxyz + xyz + yz + z$. For example, the layer sum of 4089 is $4089 + 089 + 89 + 9 = 4276$. If the layer sum of $wxyz$ equals 2014, what is the value of $w + x + y + z$?

(A) 12 (B) 15 (C) 11 (D) 13 (E) 10

Part C: Each correct answer is worth 8.

21. In the diagram, the shape consists of seven identical cubes with edge length 1. Entire faces of the cubes are attached to one another, as shown. What is the distance between P and Q ?

(A) $\sqrt{20}$ (B) $\sqrt{26}$ (C) $\sqrt{14}$
(D) $\sqrt{18}$ (E) $\sqrt{30}$



22. A five-digit positive integer is created using each of the odd digits 1, 3, 5, 7, 9 once so that
- the thousands digit is larger than the hundreds digit,
 - the thousands digit is larger than the ten thousands digit,
 - the tens digit is larger than the hundreds digit, and
 - the tens digit is larger than the units digit.

How many such five-digit positive integers are there?

- (A) 12 (B) 8 (C) 16 (D) 14 (E) 10
23. Three friends are in the park. Bob and Clarise are standing at the same spot and Abe is standing 10 m away. Bob chooses a random direction and walks in this direction until he is 10 m from Clarise. What is the probability that Bob is closer to Abe than Clarise is to Abe?
- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{\pi}$ (D) $\frac{1}{4}$ (E) $\frac{1}{6}$
24. For each positive integer n , define $S(n)$ to be the smallest positive integer divisible by each of the positive integers $1, 2, 3, \dots, n$. For example, $S(5) = 60$. How many positive integers n with $1 \leq n \leq 100$ have $S(n) = S(n+4)$?
- (A) 9 (B) 10 (C) 11 (D) 12 (E) 13
25. Point P is on the y -axis with y -coordinate greater than 0 and less than 100. A circle is drawn through P , $Q(4, 4)$ and $O(0, 0)$. How many possible positions for P are there so that the radius of this circle is an integer?
- (A) 2 (B) 68 (C) 66 (D) 65 (E) 7



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For students...

Thank you for writing the 2014 Cayley Contest!

In 2013, more than 65,000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website to find

- More information about the Galois Contest
- Free copies of past contests
- Workshops to help you prepare for future contests
- Information about our publications for mathematics enrichment and contest preparation

For teachers...

Visit our website to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Learn about our face-to-face workshops and our resources
- Find your school contest results